Summary

The aim of this experiment is the measurement of temporal and spatial correlations of different light sources. Objects of interest are a HeNe-laser and pseudothermal light, which is generated by scattering of a laser beam on a rotating ground glass disc.

Preparation

A written preparation of appropriate length is obligatory and should be done by acquainting oneself with the following catchwords. This tutorial and its drawings can be used for the preparation, it is not necessary to use other sources besides.

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1 Theoretical basics of the experiment „photon statistics“

In the following the physical and mathematical basics which are necessary for working on the experiment photon statistics are introduced.

1.1 Wave Optics

Since the propagation of light in straight rays was not able to describe all phenomena observed in experiments involving light, the development of a new theory was necessary, the wave theory. At the beginning there was the Huygens principle for the propagation of electromagnetic waves from the 17th century. In the 18th century a very successful theory was developed by T. Young and A.P. Fresnel, which combined the wave description of Huygens with the principle of interference, and was able to describe all known interference phenomena at that time. The final breakthrough was achieved by J.C. Maxwell in the middle of the 19th century with the famous Maxwell equations, which explain in an exclusively theoretical way that an electromagnetic field propagates as a transverse wave with the speed of light $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$.

The propagation of an electromagnetic wave, which consists of an electric and perpendicularly magnetic field, is described by the wave equation, which can be derived from Maxwell’s laws. In vacuum it is

$$\nabla^2 E(r, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E(r, t) = 0.$$  (1.1)

Usually complex numbers are used for the description, which simplifies most forms of waves and makes the mathematical treatment easier. Note that only the real part is of physical relevance. For a monochromatic wave with harmonic time evolution one obtains

$$E(r, t) = \Re \left[ \tilde{E}(r)e^{-i\omega t} \right],$$  (1.2)

with the angular frequency $\omega$ and the complex amplitude $\tilde{E}(r)$. In most cases it is useful to determine the real part at the end of the calculation. Therefore one writes

$$E(r, t) = \tilde{E}(r)e^{-i\omega t}.$$  (1.3)

With $\omega^2 = c^2k^2$ and the ansatz from eq. 1.3 one obtains from 1.1 the so called Helmholtz equation

$$\nabla^2 E(r) + k^2 E(r) = 0,$$  (1.4)

which depends only on the position $r$ and no longer on the time $t$. 
The most simple and important solution of the Helmholtz-equation is the plane wave. It describes a wave where all surfaces of equal phases form a bundle of planes, which are perpendicular to the direction of propagation. This means that the light source is at an infinite distance from the plane of examination. The scalar form of a plane wave is

\[ E(r, t) = E_0 e^{-i(\omega t - kr)}. \]  

(1.5)

Another important type of wave is the spherical wave, the characteristic solution of the Helmholtz equation in spherical coordinates. This solution describes light propagating isotropically, i.e. uniformly in all directions, while the amplitude decreases with \( \frac{1}{r} \)

\[ E(r, t) = \frac{E_0}{r} e^{\omega t} e^{-i(\omega t - kr)} \].

(1.6)

The superposition principle, which is fundamental for the treatment of interference phenomena, states that the field at a given point in space-time is given by the summation over all the waves \( E_j(r, t) = E_{0,j} e^{i(\omega_j t - \varphi_j)} \) in this point

\[ E(r, t) = \sum_j E_j(r, t) = \left[ \sum_j E_{0,j} e^{i\varphi_j} \right] \cdot e^{-i\omega t}, \]

(1.7)

where \( \varphi_j = kr + \phi_j \) is the phase and \( \omega_j = \omega \) \( \forall j \) in the monochromatic case. Interference takes place e.g. in Young’s double slit experiment or in a Michelson interferometer. Both experiments are described more detailed in 1.3.
1.2 Sources of Radiation

In general one distinguishes two types of sources of electromagnetic radiation. The first type are so called **classical light sources**, which are for example the sun, a gas discharge lamp and also a laser. This group includes thermal (or chaotic) sources. They are characterized by consisting of many single atoms, which emit light totally uncorrelated from each other. If one takes a closer look, such sources differ regarding their frequency spectra. The frequency spectrum of a gas discharge lamp is much more narrow than the spectrum of the sun. Anyhow they have the same photon statistic, the **Bose-Einstein distribution**. Laser light is also a classical light source, but shows a **Poisson distribution** regarding its photon statistic.

Besides of classical sources there are non-classical sources. Radiation does not occur randomly distributed, but regularly. One example is a single atom, which is excited regularly by a laser pulse and emits exactly one photon after each pulse.

1.2.1 Thermal light

Our model of a thermal source is as follows: It consists of many atoms, which emit light totally independent from each other. If atoms collide the phase changes, but the atomic state is not altered. For short interaction times during the collision processes one assumes the emitted electric field to be

\[ E(t) = E_0 e^{i(\varphi(t) - \omega t)}. \] (1.8)

That is a plane wave with amplitude \( E_0 \), angular frequency \( \omega \) and a statistically fluctuating phase \( \varphi(t) \), which is induced by the collisions (see fig. 1.1).

![Figure 1.1: (a) shows the electric field \( E(t) \) emitted by a single atom whose phase changes during the time \( t \). A collision is marked by the vertical lines and \( \tau_c \) is the mean time between collisions or in other words the mean temporal length of a wave train. (b) shows the corresponding change in phase \( \varphi(t) \).](image-url)
The radiation of a collision broadened light source is then calculated by summing over all single electric fields emitted by the atoms, which are given by eq. 1.8. For a large number of atoms and polarized light one yields

\[
E(t) = E_0 e^{-i\omega t} \left[ e^{i\varphi_1(t)} + \ldots + e^{i\varphi_n(t)} \right] = E_0 e^{-i\omega t} a(t) e^{i\varphi(t)}.
\] (1.9)

A graphical description of this summation is shown in fig. 1.2. It is a random walk with steps of equal length with randomly distributed directions (corresponding to the phases). The end point of the walk can be expressed by its distance to the origin \(a(t)\) and its phase \(\varphi(t)\).

The electric field of eq. 1.9 consists of a temporal modulation with angular frequency \(\omega\) and a randomly distributed temporal modulation of phase and amplitude. The spectrum is centered around \(\omega\) and broadened due to waves of finite length which result from the temporal modulation. Mathematically this is expressed by the presence of other frequencies than \(\omega\) in the Fourier transform of \(E(t)\). The temporal mean of the electric field is zero, but for the over one period averaged intensity \(I(t)\) one obtains

\[
I(t) = \frac{1}{T} \int_T \tilde{I}(t') dt' \propto |E(t)|^2 = E_0^2 a(t)^2.
\] (1.10)

\(I(t)\) depends only on the randomly fluctuating amplitude \(a(t)\). In the following \(I(t) = I\) is used. Figure 1.3 shows the intensity fluctuating in time. One sees that the fluctuations happen on a time scale of the magnitude of \(\tau_c\), which is the mean time between the collisions. The intensity is approximately constant on a time scale of \(\Delta t \ll \tau_c\).

The distribution function of \(I\) can be calculated from the distribution of \(a(t)\). The probability that the end point of the random walk from 1.2 lies between \(a(t)\) and \(a(t) + da(t)\) after \(n\) steps as well as \(\varphi(t)\) and \(\varphi(t) + d\varphi(t)\) is

\[
p[a(t)] da(t) d\varphi(t) = \frac{1}{\pi n} e^{-\frac{a(t)^2}{\Delta}} da(t) d\varphi(t).
\] (1.11)
1.2 Sources of Radiation

Naturally the result is independent of $\varphi(t)$. The distribution function is a Gaussian curve, which is the cause that thermal light is also called Gaussian light. Therefore, the probability distribution of $I$ is

$$p(I) = \frac{1}{\langle I(t) \rangle} e^{-\frac{I}{\langle I(t) \rangle}}$$

which is shown in fig. 1.4. Here $\langle I(t) \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} I(t) dt \propto E_0^2 n$ is the mean value of $I$ for very long time periods. In the following $\langle I \rangle$ is used instead of $\langle I(t) \rangle$. The temporal distribution of the measured intensity of a thermal source is an exponential function and is called a Boltzmann distribution. For large average photon numbers $\bar{n}$, the quantum mechanical Bose-Einstein distribution

$$p(n) = \frac{\bar{n}^n}{(1 + \bar{n})^{(1+n)}}$$

becomes identical to the Boltzmann distribution (classical limit).

The temporal length of the intensity fluctuations is given by the characteristic time $\tau_c$, which is the so-called coherence time of the source. Because of the ergodicity of the system, the spatial intensity distribution follows the same statistics.

1.2.2 Laser Light

The light of a laser, which emits only in one single mode in the ideal case, is described classically by a sine wave, which extends infinitely. The electric field can be written as

$$E(t) = E_0 e^{i(kz - \omega_0 t + \phi)},$$

where $E_0$ is the constant amplitude, $\phi$ the phase and $\omega_0$ the frequency of the wave. Classically there are no intensity fluctuations. The transverse intensity distribution can be described by a Gaussian distribution (see eq. 1.85).
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0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05

0  20  40  60  80  100  120  140

Figure 1.4: Measured intensity distribution $p(I)$ of a thermal light source with an average intensity $\langle I \rangle = 20 \text{ a.u.}$

1.2.3 Pseudothermal Light

Because of the very short coherence times of real thermal light sources one often uses so-called pseudothermal (or quasithermal) sources in experiments.

If coherent light is scattered on an optical rough surface many single independent waves with independent phases are generated. The different phases result from the different optical path lengths, which have to be passed by the initially coherent light. The superposition of these waves leads to a spatially varying intensity distribution in the far field. This intensity distribution is called a speckle pattern. The speckle phenomenon can also be observed with only partially spatial coherent radiation like the light from the sun or the stars.

One sees bright and dark areas in the speckle pattern of fig. 1.5. It can also be described with the model of a random walk for the electric field. The corresponding spatial intensity distribution is described by the Boltzmann statistics

$$p(I) = \frac{1}{\langle I \rangle} e^{-\frac{I}{\langle I \rangle}},$$

(1.15)

with an average intensity $\langle I \rangle$. In order to generate pseudothermal radiation, a ground glass disc is illuminated by coherent laser light. By rotating the ground glass disc, the position of the spot on the glass is changed and thus the speckle pattern varies. The resulting intensity fluctuations take place in a temporal regime which can be easily resolved by usual measurement tools. That is the reason for the usage of a pseudothermal source in most experiments dealing with thermal light. A typical coherence time of a gas discharge lamp is in the order of $< 10^{-9} \text{ s}$ and thus detection of fluctuations can only be done with some experimental efforts.
Figure 1.5: A speckle pattern grabbed by a CCD camera in a distance of 20 cm from a ground glass disc which was illuminated by a HeNe-laser.

Conclusion:

\[
\text{coherent laser light + rotating ground glass disc} = \text{pseudothermal light}
\]

The average diameter of a speckle on a observing screen in a distance \( z \) from the source is

\[
d_{\text{sp}} = \frac{\lambda z}{2\omega_0},
\]

(1.16)

where \( \lambda \) is the wavelength of the laser and \( \omega_0 \) is the radius of the beam at the ground glass disc.

Within a speckle, the light is spatially coherent, so a speckle spot can be called a coherence cell. However, two or more speckles are totally uncorrelated to each other. By altering the rotation speed of the ground glass disc, the velocity with which the speckle pattern varies can be adjusted. It appears that the statistics of the distribution of the intensities in the static as well as in the dynamic case is the same as the statistics of a real thermal light source. The corresponding coherence time is much longer and can be regulated by the rotation speed of the disc. The adjustable range lies within 1 \( \mu s - 1 \) ms. The coherence time \( \tau_c \) can be calculated by

\[
\tau_c = \frac{\omega_0}{v} = \frac{\omega_0}{2\sqrt{\pi v} \nu},
\]

(1.17)

where \( v \) is the tangential speed of the disk at the point where the laser light impinges on the disk. It can be determined if one knows the rotation frequency of the disc \( \nu \) and the distance from the laser point to the rotational axis of the disc \( r \). This equation only holds for a Gaussian beam profile of a laser in the TEM\(_{00}\)-mode.
1.3 Coherence

The term coherence plays an important role in many fields of modern physics. Some examples from the field of optics are coherent light, coherent states or coherent scattering. A general definition of coherence is the following:

*A process is coherent, if it is characterized by a well-defined, deterministic phase relation, i.e. there are no random fluctuations of the phase.*

In this experiment the coherence properties of different light sources are investigated. It is convenient to differentiate between temporal and spatial coherence. The first term refers to the finite spectral bandwidth of the source, the second term to the finite geometrical extension of a phase relation of the wave in space.

We assume a quasi-monochromatic source as a series of finite wave trains, whose frequency and amplitude varies only a bit around a mean frequency $\bar{\nu}$ and amplitude $\bar{A}$. Such a wave train exists approximately for the coherence time $\tau_c$, which is inversely proportional to the spectral bandwidth of the source $\Delta \nu$. The bandwidth of an ideal monochromatic source would be described by a $\delta$-distribution and the coherence time would be infinite. For a quasi-monochromatic light source $\Delta \nu$ is finite and hence $\tau_c$ is finite, too. Within a time interval smaller than $\tau_c$ the source emits light with a fixed phase relation. Thus, the coherence time can also be described as the *temporal interval, within which the phase of a lightwave at a given point can be predicted.* (see fig. 1.6). The spatial length which corresponds to this temporal interval is called *longitudinal coherence length* and is calculated by

$$b_c = c \cdot \tau_c.$$  \hspace{1cm} (1.18)

![Figure 1.6: Temporal coherence](image)

(a) A monochromatic light source $L$ emits wave trains which are coherent over an arbitrary distance $P_1P_2 = \Delta d$. (b) A quasi-monochromatic light source $L$ alters its frequency. The wave trains at $P_1$ and $P_2$ are not coherent, if the longitudinal coherence length $b_c$ is smaller than $\Delta d$.

Spatial coherence refers to the correlations of an extended light source between two space points. If those two points lie on the same wave front, their fields are coherent at these points. In order to observe interference between two light waves, they have to be coherent to each other. One example for that is the double slit experiment with an extended source of
lateral width $\Delta s$ from Thomas Young (see fig. 1.7). An interference pattern in $r$ can only be observed if the fields at $r_1$ and $r_2$ in the plane $A$ are coherent. We assume that the slits have both the same distance from the light source, so that the fields are temporally coherent. In addition to that the fields have to be spatially coherent, which means that they have to lie within the coherence area $\Delta A$ of the source, which is inversely proportional to the lateral width of the source

$$\Delta A \propto \frac{1}{\Delta s} \quad (1.19)$$

The square root of the coherence area is called transverse coherence length. This relation clearly explains the necessity of a slit to achieve spatial coherence for interference experiments dealing with extended light sources. For a better understanding see fig. 1.7 and its description.

Figure 1.7: A thermal, quasi-monochromatic source with lateral extension $\Delta s$ illuminating two small slits at $r_1$ and $r_2$. The fluctuations at different spatial positions of the source are totally independent of each other, so that the intensity pattern in the plane $B$ results from the addition of the different independent patterns. If one increases the distance between the slits $|r_1 - r_2|$, the maxima of the single patterns move apart. At distances between the slits, which exceed the transverse coherence length of the source, the interference pattern blurs completely and is not longer visible.
1.4 Correlations of First Order

Now we want to take a closer look at Young’s double slit experiment (see fig. 1.7), where two slits are illuminated with coherent light and thus an interference pattern results in the observations plane $B$. The electric field $\hat{E}(r,t)$ at $r$ is formed as a superposition of the fields $E_1(r,t)$ and $E_2(r,t)$, which originate from the slits at $r_1$ and $r_2$.

\[
\hat{E}(r,t) = E_1(r,t) + E_2(r,t) = E_1\left( r_1, t - \frac{r - r_1}{c} \right) + E_2\left( r_2, t - \frac{r - r_2}{c} \right),
\]

(1.20)

where $\tau = \frac{|r_2 - r_1| - |r_1 - r|}{c}$ indicates the run-time difference between the rays from the different slits to $r$. Therefore, the intensity at $r$ is

\[
I(r,t) \propto \left| \hat{E}(r,t) \right|^2 = |E_1(r_1,t)|^2 + |E_2(r_2,t + \tau)|^2 + 2\Re[E_1^*(r_1,t)E_2(r_2,t + \tau)].
\]

(1.21)

Because of technical limits one cannot measure $I(r,t)$ with arbitrary precision. Instead, one measures a mean value over a temporal interval. In the following $\langle \ldots \rangle$ indicates the mean over the period $T$

\[
\langle I(r,t) \rangle \propto \langle \left| \hat{E}(r,t) \right|^2 \rangle = \langle |E_1(r_1,t)|^2 \rangle + \langle |E_2(r_2,t + \tau)|^2 \rangle + 2\Re[\langle E_1^*(r_1,t)E_2(r_2,t + \tau) \rangle].
\]

(1.22)

The first two terms correspond to the intensity of each slit, which would appear in the absence of the other. The latter term describes the correlation of both electric fields. We assume equal amplitudes $E_1(r_1,t) = E_2(r_2,t + \tau)$, so that one gets the intensity correlation function of first order from this term by normalizing it

\[
g^{(1)}(r_1,r_2,\tau) = \frac{\langle E_1^*(r_1,t)E_2(r_2,t + \tau) \rangle}{\langle E_1^*(r_1,t)E_1(r_1,t) \rangle}.
\]

(1.23)

If one measures the correlation function $g^{(1)}(r_1,r_2,\tau)$ of two fields at the same position $r_1 = r_2$, one obtains the temporal correlation function of first order. In the following, it will be written in the short notation $g^{(1)}(\tau)$. If one measures $g^{(1)}(r_1,r_2,\tau)$ at distinct positions, but at the same time, which means $\tau = 0$, one gets the spatial correlation function on first order, which will be indicated by $g^{(1)}(r_1,r_2)$.

Another interference experiment which can be described in terms of correlations is the Michelson-interferometer (see fig. 1.8). In this experiment a beam of light is divided into two beams using a beam splitter. Both beams are then reflected back on the beam splitter with the mirrors $S_1$ and $S_2$, which introduces a path difference $2\Delta s$, so that there is a run time difference of $\tau = \frac{2\Delta s}{c}$ between the beams.

Both beams are superposed after the beam splitter and a interference pattern can be observed in the plane $B$, but only if the run time difference is smaller than the coherence time of the observed light. If the run time difference is larger, there are no correlations of the field amplitudes and no interference pattern results. With this experiment the coherence time $\tau_c$ of a light source can be determined.
1.4 Correlations of First Order

Figure 1.8: Light from a source $L$ is divided by a beam splitter. After that each beam is reflected by the mirrors $S_1$ and $S_2$ back onto the beam splitter. A superposition of the two beams which have a path difference of $2\Delta s$ is generated and thus an interference pattern is observed in the plane $B$.

The visibility $V$ of an interference pattern is defined by

$$V = \frac{\langle I \rangle_{\text{max}} - \langle I \rangle_{\text{min}}}{\langle I \rangle_{\text{max}} + \langle I \rangle_{\text{min}}}.$$  \hfill (1.24)

If the visibility is maximal, which is the case if $\langle I \rangle_{\text{min}} = 0$, the impinging light field is fully coherent.

The interference term from eq. 1.22 oscillates between $\pm 2|\langle E_1^*(r_1, t)E_2(r_2, t + \tau) \rangle|$, which results in a visibility

$$V = \bigg| g^{(1)}(r_1, r_2, \tau) \bigg|.$$  \hfill (1.25)

The visibility lies between 0 and 1, therefore the light is incoherent in the first case and coherent in the latter one. For values between one talks about partial coherence.

In measurements of the correlation function of first order, field amplitudes of an electromagnetic field at different space time points are correlated. If a light source is coherent in first order, interference effects can be observed.

1.4.1 Wiener-Khintchine Theorem

As mentioned in sec. 1.3, there is a relation between the temporal correlation function of first order $g^{(1)}(\tau)$ and the spectral intensity distribution of a light source, which is known as the Wiener-Khintchine theorem.

The power spectrum is calculated from the Fourier transform of the amplitude

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} E(t)e^{i\omega t}dt,$$  \hfill (1.26)
from which the intensity can be determined

\[
|E(\omega)|^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E^*(t) E(t') e^{i\omega(t'-t)} dt dt' \\
= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E^*(t) E(t + \tau) e^{i\omega \tau} d\tau \text{ mit } \tau = t' - t.
\] (1.27)

With \( \langle E^*(t) E(t + \tau) \rangle = \frac{T}{T} \int T E^*(t) E(t + \tau) d\tau \) the equation for stationary fields yields

\[
|E(\omega)|^2 = \frac{T}{2\pi} \int_{-\infty}^{+\infty} \langle E^*(t) E(t + \tau) \rangle e^{i\omega \tau} d\tau. \tag{1.28}
\]

By normalizing on the intensity one obtains the spectral intensity distribution

\[
F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g^{(1)}(\tau) e^{i\omega \tau} d\tau. \tag{1.29}
\]

\( F(\omega) \) is the Fourier transform of the temporal correlation function of first order \( g^{(1)}(\tau) \) and the spectral width is inversely proportional to the coherence time \( \tau_c \). Note that the frequency spectrum of the light source, which is often a Gaussian or Lorentzian, must not be mistaken with the Gaussian distribution of \( |E(t)| \) from sec. 1.2.1!

### 1.4.2 Collision broadened thermal light source

The model introduced in sec. 1.2.1 will now be used to calculate the correlation function of first order for thermal light. The summation over all terms of the \( n \) atoms yields

\[
\langle E^*(t) E(t + \tau) \rangle = E_0^2 e^{-i\omega \tau} \left( e^{-i\varphi_1(t)} + \ldots + e^{-i\varphi_n(t)} \right) \\
\times \left( e^{i\varphi_1(t+\tau)} + \ldots + e^{i\varphi_n(t+\tau)} \right) \tag{1.30}
\]

The product of the phase terms vanishes through the averaging, since single phases are distributed statistically random. Therefore one obtains

\[
\langle E^*(t) E(t + \tau) \rangle = E_0^2 e^{-i\omega \tau} \sum_{j=1}^{n} \left( e^{i\varphi_j(t+\tau)} e^{-i\varphi_j(t)} \right) \\
= n \langle E^*_j(t) E_j(t + \tau) \rangle, \tag{1.31}
\]

since the field of all atoms are equivalent. The correlation function of the whole source depends on the contributions of the single atoms. The one-atom correlation function is proportional to the probability for one atom emitting a wave train of length \( > \tau \)

\[
\langle E^*_j(t) E_j(t + \tau) \rangle = E_0^2 e^{-i\omega \tau} \int_{\tau}^{\infty} p(t) dt. \tag{1.32}
\]

whereat \( p(t) dt = \frac{1}{\tau_c} e^{-\frac{t}{\tau_c}} dt \) is the probability for the time between two collision to lie within the interval \([t, t + dt]\). The characteristic time between two collision is called coherence time \( \tau_c \). It follows
\[ \langle E_j^* (t) E_j (t + \tau) \rangle = E_0^2 e^{\left( -i\omega_0 - \frac{i}{\tau_c} \right) \tau}. \]  \tag{1.33}

The temporal correlation function of first order yields
\[ g^{(1)} (\tau) = e^{-i\omega_0 \tau - \frac{1}{\tau_c} |\tau|}. \]  \tag{1.34}

From that one can calculate the spectral intensity distribution using the Wiener-Khintchine theorem
\[ F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g^{(1)} (\tau) e^{i\omega \tau} d\tau = \frac{1/\tau_c}{(\omega - \omega_0)^2 + (1/\tau_c)^2}. \]  \tag{1.35}

The frequency spectrum of a collision broadened light source has a Lorentzian shape.

Figure 1.9: \[ |g^{(1)} (\tau)| \] of a light source with Lorentz-shaped frequency spectrum.

### 1.4.3 Doppler-broadened thermal light source

Another type of thermal light is generated by sources whose line-broadening is caused by the Doppler-effect. Different emitting atoms contribute with Doppler-shifted frequencies \( \omega_j \) to the whole field
\[ E (t) = E_0 \sum_{j=1}^{n} e^{-i(\omega_j t + \varphi_j)}. \]  \tag{1.36}

\( E_0 \) are the constant field amplitudes and \( \varphi_j \) is the phase of the radiation emitted by the \( j \)th atom. As cross-correlation one obtains
\[ \langle E^* (t) E (t + \tau) \rangle = E_0^2 \sum_{j,k=1}^{n} \left| \langle e^{i\omega_j t - i\varphi_j - i\omega_k (t+\tau) + i\varphi_k} \rangle \right|. \]  \tag{1.37}

The contributions for \( j \neq k \) vanish by averaging due to the statistical independence of the single fields
\[ \langle E^* (t) E (t + \tau) \rangle = E_0^2 \sum_{j=1}^{n} e^{-i\omega_j \tau}. \] (1.38)

For a large number of atoms the sum can be written as an integral over a Gaussian distribution of the Doppler-shifted frequencies with a width \( \delta = \frac{1}{\tau c} \).

\[ \langle E^* (t) E (t + \tau) \rangle = \frac{nE_0^2}{\sqrt{2\pi} \delta} \int_{0}^{\infty} e^{-i\omega \tau - \frac{(\omega - \omega_0)^2}{2\pi \delta^2}} d\omega = nE_0^2 e^{-i\omega_0 \tau - \frac{\pi \delta^2 \tau^2}{2}}, \] (1.39)

where \( \omega_j = \omega \forall j \).

The temporal correlation function of first order yields

\[ g^{(1)} (\tau) = e^{-i\omega_0 \tau - \frac{\pi \delta^2 \tau^2}{2}} \] (1.40)

with \( \delta = \frac{1}{\tau c} \). The absolute value of \( g^{(1)} (\tau) \) is a Gaussian curve.

From \( g^{(1)} (\tau) \) one can again calculate the spectral intensity distribution using the Wiener-Khintchine theorem:

\[ F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g^{(1)} (\tau) e^{i\omega \tau} d\tau = \frac{\tau c}{\sqrt{2\pi}} e^{-(\omega - \omega_0)^2 \tau^2}. \] (1.41)

The frequency spectrum of a Doppler-broadened light source has a Gaussian shape.

Figure 1.10: \( |g^{(1)} (\tau)| \) of a light source with a Gaussian shaped frequency spectrum.
1.4 Correlations of First Order

1.4.4 Laser Light

In section 1.2.2 the electrical field of a laser beam was introduced as a sinusoidal oscillation with a fixed phase $\phi$. In this case the correlation function is

$$\langle E^* (t) E (t + \tau) \rangle = E_0^2 e^{-i\omega_0 \tau}. \quad (1.42)$$

The temporal correlation function of first order then becomes

$$g^{(1)} (\tau) = e^{-i\omega_0 \tau}. \quad (1.43)$$

That means $|g^{(1)} (\tau)| = 1$ for all $\tau$. In the classical description laser light has no fluctuations of the field amplitude and is thus called coherent (in first order). This result can be obtained by altering the line width of a broadened thermal light source towards zero, i.e. considering a light source with infinite coherence time $\tau_c$. 

Figure 1.11: Comparison of $F(\omega)$ of a Doppler-broadened and a collision broadened thermal light source with $\tau_c = 1$ s.
1.5 Intensity Correlations of Second Order

Analogously to \( g^{(1)}(r_1, r_2, \tau) \) one defines the normalized intensity correlation function of second order as follows:

\[
g^{(2)}(r_1, r_2, \tau) = \frac{\langle E^*(r_1, t)E^*(r_2, t + \tau)E(r_2, t + \tau)E(r_1, t) \rangle}{\langle E^*(r_1, t)E(r_1, t) \rangle \langle E^*(r_2, t)E(r_2, t) \rangle}
\]

\[
= \frac{\langle I(r_1, t)I(r_2, t + \tau) \rangle}{\langle I(r_1, t) \rangle \langle I(r_2, t) \rangle}.
\]

The difference between the correlation functions of different orders is that in one case two field amplitudes are correlated and in the other case four amplitudes, which correspond to two intensities. Here also \( g^{(2)}(r_1, r_2 = r_1, \tau) = g^{(2)}(\tau) \) is called temporal correlation function of second order and \( g^{(2)}(r_1, r_2, \tau = 0) = g^{(2)}(r_1, r_2) \) is the spatial correlation function of second order.

Light is called coherent in second order if

\[
\left| g^{(1)}(r_1, r_2, \tau) \right| = 1 \quad \text{and} \quad g^{(2)}(r_1, r_2, \tau) = 1.
\]

is valid. This is true e.g. for laser light. Thus, in the classical description laser light is coherent in first and second order.

From \( g^{(2)}(\tau = 0) \) one can deduce information about the intensity fluctuations of a light source. In this special one obtains

\[
g^{(2)}(0) = \frac{\langle I^2(t) \rangle}{\langle I(t) \rangle^2} = 1 + \frac{\langle \Delta I^2(t) \rangle}{\langle I(t) \rangle^2},
\]

where \( \Delta I(t) = I(t) - \langle I(t) \rangle \). In the quantum theory of light, intensities are interpreted as photon numbers. Because of that conclusions about the photon statistics can be deduced from \( g^{(2)}(0) \). This will be explained more detailed in sec. 1.7.3

1.5.1 Hanbury Brown and Twiss Experiment

In the Hanbury Brown and Twiss (HBT) experiment, the coherence of second order, i.e. the correlations of intensities, are investigated. Because of that it is sometimes referred to as an intensity interferometer. There are two types of this experiment. One version is for the measurement of temporal correlations of second order, the other for the observation of spatial correlations of second order.

The experimental setup shown in fig. 1.12 can be used to determine the temporal coherence of a light source. A beam of light is divided into two beams whose intensities are measured by the detectors \( D_1 \) and \( D_2 \). The signal of detector \( D_2 \) is delayed by the time \( \tau \) and then both signals are correlated electrically, so that \( (I_1(t)I_2(t + \tau)) \) is measured. One can alter this setup in a way that \( \tau = 0 \) and detector \( D_2 \) can be moved perpendicular to the optical axis (see 1.13). By doing so the spatial correlations of second order \( g^{(2)}(r_1, r_2) \) can be measured.

The measurement of the spatial intensity correlations was used by Hanbury Brown and Twiss for the determination of the angular diameter of stars. A great advantage compared to Michelson-type stellar interferometers, which use amplitude interferences, is the independence from phase fluctuations, since only the intensity is measured. By assuming a circular star of diameter \( a \) with a homogenous intensity distribution in a great distance from the earth, one can calculate the analytic expression of \( g^{(2)}(r_1, r_2) \) using the van
1.5 Intensity Correlations of Second Order

Figure 1.12: Experimental setup of the HBT experiment for the measurement of the temporal correlation function of second order. A light beam from the source $L$ is divided and each intensity is measured by the detectors $D_1$ and $D_2$. The signal of $D_2$ is delayed by the time $\tau$ before the two signals are correlated.

Figure 1.13: Sketch of the HBT experiment for the measurement of the spatial coherence of second order of a light source. Light from the source $L$ is divided with a beam splitter cube and the intensity of each beam measured with the detectors $D_1$ at $r_1$ and $D_2$ at $r_2$. The detector $D_2$ can be moved lateral, so that by correlation of the signals $g^{(2)}(r_1, r_2)$ can be determined.

**Cittert-Zernike theorem.** From this the diameter of the star can be determined, if the distance from the earth to the star is known.

With the HBT intensity interferometer from fig. 1.14 light from an extended distant source is collected using two mirrors, whose distance $d$ can be varied. The light is focused on two photomultipliers which measure the intensities, afterwards their signals are correlated. With the help of the Siegert relation and the van Cittert-Zernike theorem the geometry of the source can be determined.
1.5.2 Siegert relation

In the following the derivation of the Siegert relation is sketched. This important relation is only valid for thermal light and builds a link between $g^{(2)}(r_1, r_2, \tau)$ and $g^{(1)}(r_1, r_2, \tau)$. In a thermal light field $j$ independent atoms contribute to the entire field

$$E(t) = \sum_{j=1}^{n} E_j(t).$$

(1.48)

The counter from eq. 1.44 then yields
1.5 Intensity Correlations of Second Order

\[ \langle E^*(r_1,t)E^*(r_2,t+\tau)E(r_2,t+\tau)E(r_1,t) \rangle \]

\[ = \sum_{j=1}^{n} \langle E_j^*(r_1,t)E_j^*(r_2,t+\tau)E_j(r_2,t+\tau)E_j(r_1,t) \rangle \]

\[ + \sum_{j \neq k} \left[ \langle E_j^*(r_1,t)E_k^*(r_2,t+\tau)E_k(r_2,t+\tau)E_j(r_1,t) \rangle \right] \]

\[ + \langle E_j^*(r_1,t)E_j^*(r_2,t+\tau)E_j(r_2,t+\tau)E_k(r_1,t) \rangle \].

All atoms are equivalent and only the terms whose fields are multiplied with its complex conjugate give a contribution.

\[ \langle E^*(r_1,t)E^*(r_2,t+\tau)E(r_2,t+\tau)E(r_1,t) \rangle \]

\[ = n \langle E_j^*(r_1,t)E_j^*(r_2,t+\tau)E_j(r_2,t+\tau)E_j(r_1,t) \rangle \]

\[ + n(n-1) \left[ \langle E_j^*(r_1,t)E_j(r_1,t) \rangle \langle E_j^*(r_2,t+\tau)E_j(r_2,t+\tau) \rangle \right] \]

\[ + n(n-1) \left[ \langle E_j^*(r_1,t)E_j(r_2,t+\tau) \rangle \langle E_j^*(r_2,t+\tau)E_j(r_1,t) \rangle \right] \].

For a great number of atoms, all terms proportional to \( n \) can be neglected.

\[ \langle E^*(r_1,t)E^*(r_2,t+\tau)E(r_2,t+\tau)E(r_1,t) \rangle \]

\[ = n^2 \left[ \langle E_j^*(r_1,t)E_j(r_1,t) \rangle \langle E_j^*(r_2,t+\tau)E_j(r_2,t+\tau) \rangle \right] \]

\[ + n^2 \left[ \langle E_j^*(r_1,t)E_j(r_2,t+\tau) \rangle \langle E_j^*(r_2,t+\tau)E_j(r_1,t) \rangle \right] . \]

By inserting this into eq. 1.44 and using eq. 1.23 under the assumption of equal amplitudes one obtains the important **Siegert relation** for thermal light

\[ g^{(2)}(r_1,r_2,\tau) = 1 + \left| g^{(1)}(r_1,r_2,\tau) \right|^2 . \]  

(1.52)

It states that for thermal light the correlation function of second order \( g^{(2)}(r_1,r_2,\tau) \) can be expressed by the correlation function of first kind \( g^{(1)}(r_1,r_2,\tau) \). Particularly for the temporal correlation function of second order the following is valid:

\[ g^{(2)}(\tau) = 1 + \left| g^{(1)}(\tau) \right|^2 . \]  

(1.53)

In fig. 1.15 the results are illustrated. The maximum of \( g^{(2)}(\tau) \) for \( \tau < \tau_c \) originates from intensity fluctuations, as shown in fig. 1.3. For very short delay times, both intensity measurements lie within the same fluctuation, which leads to an increased correlation. In the photon concept, this can be interpreted as follows: Light particles have a tendency to impinge on the detector in bundles. This effect is known as photon bunching. For long delay times \( \tau \), the correlation between the intensities declines. In the limiting case \( \tau \to \infty \) the intensities are statistically independent and one obtains

\[ g^{(2)}(\tau) = \frac{\langle I(t) \rangle \langle I(t+\tau) \rangle}{\langle I(t) \rangle^2} = 1. \]  

(1.54)

This corresponds to randomly appearing correlations between both light beams. The line broadening effect of the source influences not only the electric field, but also the intensity of the source, which varies around a mean value. The time scale \( \tau_c \) of these fluctuations is inversely proportional to the spectral width of the light.
1.5.3 Van Cittert-Zernike Theorem

The van Cittert-Zernike theorem describes the relation between the spatial intensity distribution of an extended incoherent light source and the first-order spatial correlation function \( g^{(1)}(r_1, r_2) \)

\[
g^{(1)}(r_1, r_2) = e^{ik(r_2-r_1)} \frac{\int_\sigma I(r') e^{-ik\left[|s_2-s_1|r\right]} d^2r'}{\int_\sigma I(r') d^2r'},
\]  

(1.55)

where \( r_j = |r_j| \) is the distance of the source \( L \) to \( r_j = r_j s_j \) with the unit vector \( s_j \) in the direction to \( r_j \). \( \sigma \) is the geometry of the source \( L \) with the intensity distribution \( I(r') \). \( g^{(1)}(r_1, r_2) \) is proportional to the two-dimensional Fourier transform of the intensity distribution \( I(r') \) over the domain \( \sigma \).

Therefore the geometry of the source can be inferred from the first-order correlation function. Fig. 1.16(a) shows a homogeneous, quasimonochromatic and incoherent source \( L \) with an intensity distribution \( I(r') \) over the domain \( \sigma \). If \( r_1 \) is fixed in the detection plane and \( r_2 \) is varied over the detection plane, \( g^{(1)}(r_1, r_2) \) can be calculated via eq. 1.55. \( g^{(1)}(r_1, r_2) \) cannot be measured directly in the plane \( B \). For incoherent sources, it can be calculated from \( g^{(2)}(r_1, r_2) \) via the Siegert relation (cf. fig. 1.16(c) and eq. 1.57). Therefore \( g^{(2)}(r_1, r_2) \) allows to infer the source geometry via the van Cittert-Zernike theorem.

The functional form of \( g^{(1)}(r_1, r_2) \) for an incoherent source in eq. 1.55 is the same as the Fraunhofer diffraction pattern for a coherent source with the same geometry.

For a slit aperture \( g^{(1)}(r_1, r_2) \) is described by a sinc-function \( \left( \frac{\sin(x)}{x} \right) \), while a circular aperture causes an Airy pattern

\[
g^{(1)}(r_1, r_2) = \frac{2J_1(X)}{X},
\]  

(1.56)
where $X = \frac{\pi a (r_2 - r_1)}{R X}$, with the diameter $a$ of the aperture and the distance from source to detector $R$. $J_1$ is the first-order Bessel function. Using the Siegert-relation one obtains for $g^{(2)}$:

$$g^{(2)}(r_1, r_2) = 1 + \left| \frac{2J_1(X)}{X} \right|^2.$$  

(1.57)
Figure 1.16: (a) Geometry of the van Cittert-Zernike Theorems. \( L \) is a quasimonochromatic, incoherent source with the geometry \( \sigma \), \( B \) is the detection plane and the points \( P_1 \) and \( P_2 \) are defined by \( r_1 \) respectively \( r_2 \). (b) Spatial first-order correlation function \( g^{(1)}(r_1, r_2) = \frac{2J_1(X)}{X} \) as a function of the distance between \( r_1 \) and \( r_2 \) for an incoherently illuminated circular aperture. \( g^{(1)}(r_1, r_2) \) cannot be measured directly and corresponds to the normalized diffraction pattern of a coherently illuminated circular aperture. (c) \( g^{(2)}(r_1, r_2) = 1 + \left| \frac{2J_1(X)}{X} \right|^2 \) for a circular aperture. \( D_1 \) and \( D_2 \) are the detectors.
1.6 Basic Quantum Description

In the following, an overview over the quantum description of the electromagnetic field is given. The electromagnetic field is quantized by identifying every mode of the field, defined by the wave vector $k$, with a harmonic oscillator. The spatial distribution of the modes $k$ is neglected in the following.

The Hamiltonian of the harmonic oscillator is given by

$$\hat{H} = \frac{1}{2} \left( \hat{p}^2 + \omega^2 \hat{q}^2 \right), \quad (1.58)$$

with the frequency $\omega$, the position operator $\hat{q}$ and the momentum operator $\hat{p}$, which follow the commutator rule $[\hat{q}, \hat{p}] = i\hbar$.

Commonly, $\hat{p}$ and $\hat{q}$ are replaced by the dimensionless creation and annihilation operators:

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar\omega}} (\omega \hat{q} - i \hat{p}), \quad \hat{a} = \frac{1}{\sqrt{2\hbar\omega}} (\omega \hat{q} + i \hat{p}).$$

Therefore eq. 1.58 can be written as

$$\hat{H} = \frac{1}{2} \hbar \omega \left( \hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{a} \right) = \hbar \omega \left( \hat{a} \hat{a}^\dagger + \frac{1}{2} \right). \quad (1.59)$$

This is the Hamiltonian of the electromagnetic field.

The Eigenstates of $\hat{H}$ are the number states $|n\rangle$ with the energy $E_n = \hbar \omega \left( n + \frac{1}{2} \right)$ known from the harmonic oscillator. An electromagnetic field in the state $|n\rangle$ contains $n$ energy quanta which are called photons. The operator $\hat{a}$ annihilates a photon, the operator $\hat{a}^\dagger$ creates one.

$$\hat{a} |n\rangle = \sqrt{n} |n - 1\rangle, \quad \hat{a}^\dagger |n\rangle = \sqrt{n + 1} |n + 1\rangle. \quad (1.60)$$

The annihilation and creation operator follow the commutation rule for bosons:

$$[\hat{a}, \hat{a}^\dagger] = 1. \quad (1.61)$$

The intensity of the field is given by the expectation value of the photon number operator which is equivalent to the mean number of photons in the mode.

$$\langle \hat{I} \rangle \propto \langle \hat{n} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle \equiv \bar{n}. \quad (1.62)$$

The quantum mechanical temporal second-order correlation function can be written as a function of annihilation and creation operators:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle^2}. \quad (1.63)$$

In contrast to the classical description, the order of the operators, which correspond to the classical fields, is important (cf. eq. 1.61).

Using eq. 1.61 and 1.62 $g^{(2)}(0)$ can be written as a function of the expectation value of the photon number operator $\bar{n}$:

$$g^{(2)}(0) = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2} = 1 + \frac{\Delta n^2 - \bar{n}}{\bar{n}^2}, \quad (1.64)$$
where $\Delta n^2 = \langle \hat{n} - \langle \hat{n} \rangle \rangle^2$.

Analogous to $I(t) \propto \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle$ which gives the probability to detect a photon at the time $t$, $g^{(2)}(\tau)$ can be interpreted as a conditional probability:

$$g^{(2)}(\tau) = \frac{P(t|t + \tau)}{P(t)}.$$  \hspace{1cm} (1.65)

$P(t)$ is the probability to detect a photon at the time $t$ and $P(t|t + \tau)$ the conditional probability to detect a second photon at the time $t + \tau$ if the first photon was detected. This interpretation of $g^{(2)}(\tau)$ requires the temporal order of the operators.

Hence the spatial correlation function can be written as

$$g^{(2)}(r_1, r_2) = \frac{P(r_2|r_1)}{P(r_2)},$$  \hspace{1cm} (1.66)

where $P(r_2)$ is the probability to detect a photon at the position $r_2$ and $P(r_2|r_1)$ the conditional probability to detect a second photon at $r_2$ if a photon was detected at $r_1$. 
1.7 Photon Statistics

Photon statistics describe the probability distribution $p(n, T)$, to detect $n$ photons in the temporal interval $T$. First, the photon statistics for a Laser source is derived, in the next paragraph for a thermal source. Afterwards, these statistics are compared with each other.

1.7.1 Photon Statistics of a Laser

Quantum mechanically, the monochromatic field of a single-mode Laser is described by the coherent states $|\alpha\rangle$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$  

(1.67)

Therefore, for the intensity holds:

$$\langle I \rangle \sim \langle \hat{n} \rangle = \langle \alpha | \hat{a}^\dagger \hat{a} | \alpha \rangle = |\alpha|^2 = \bar{n}.$$  

(1.68)

The probability to measure a given photon number is given by the projection

$$p(n) = |\langle n | \alpha \rangle|^2 = \frac{|\alpha|^2 n!}{n!} e^{-|\alpha|^2} = \frac{n!}{n!} e^{-\bar{n}},$$  

(1.69)

which is a Poisson-distribution with the mean value $\bar{n}$.

For the variance of the photon number one finds

$$\Delta n^2 = \bar{n}.$$  

(1.70)

On the contrary to the classical description, the Laser light shows fluctuations. These result from the particle character of the light (shot noise). For large photon numbers, the relative fluctuations $\frac{\Delta n^2}{\bar{n}^2}$ tend to zero. Therefore, the classical theory is contained as a limiting case.

1.7.2 Photon Statistics of a Thermal Source

In the following, a thermal light source which oscillates on a single mode with the frequency $\omega$ is discussed. If the field is in the thermal equilibrium at the temperature $T$ one gets a photon number distribution, which corresponds to a normalized Boltzmann distribution

$$p(n) = \left(1 - e^{-\frac{\hbar \omega}{k_B T}}\right) e^{-\frac{n \hbar \omega}{k_B T}}.$$  

(1.71)

Expressed by the mean photon number $\bar{n}$ from eq. 1.62 the distribution can be written as

$$p(n) = \frac{\bar{n}^n}{(1 + \bar{n})^{1+n}},$$  

(1.72)

which is a Bose-Einstein distribution with the mean value $\langle I \rangle \propto \bar{n}$.

For the variance of the photon number one obtains

$$\Delta n^2 = \bar{n}^2 + \bar{n} \quad \text{for} \quad T \ll \tau_c.$$  

(1.73)

For $\bar{n} \gg 1$ the second term can be neglected. On the contrary to the Laser, for large photon numbers the relative fluctuation $\frac{\Delta n^2}{\bar{n}^2}$ do not tend to 0 but to 1. Further, for large values of $n$ eq. 1.72 can be written as

$$p(n) \approx \frac{1}{\bar{n}} e^{-\frac{n}{\bar{n}}},$$  

(1.74)
This closely resembles eq. 1.15. If $T > \tau_c$, the statistics becomes Poissonian, because photons which do not belong to the same fluctuation and are therefore uncorrelated are included. The same would be observed if the detector area would exceed the spatial coherence area, because the different fluctuations would average.

Figure 1.17: Poisson distribution of a Laser and Bose-Einstein distribution of a thermal light source, both with $\bar{n} = 15$.

### 1.7.3 Bunching

The phenomenon that photons from a thermal source are emitted in „bunches“, the so-called *Bunching*, causes intensity fluctuations. If a photon was detected, the probability to detect a second photon directly afterwards is increased. This can be investigated using $g^{(2)}(\tau)$ (cf. eq. 1.44). The intensity fluctuations can be written as

$$\Delta I(t) = I(t) - \langle I(t) \rangle.$$  

(1.75)

using this, eq. 1.44 can be written as

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2} = 1 + \frac{\langle \Delta I(t)\Delta I(t+\tau) \rangle}{\langle I(t) \rangle^2}.$$  

(1.76)

One can see that a source whose intensity fluctuations tend to 0 are coherent in second order. An example is Laser light in classical description. On the contrary a thermal source shows increased correlations for small values of $\tau$ which goes along with an increased probability to detect a second photon after the detection of a first one. This increased correlation vanishes for larger $\tau$. From this it can be inferred that $g^{(2)}$ has a maximum at $\tau = 0$ and decreases asymptotically to 1.

From eq. 1.76 at $\tau = 0$ the intensity fluctuations can be obtained. In the quantum description of light, intensities are interpreted as photon numbers, therefore $g^{(2)}(0)$ allows to infer the underlying photon statistics. According to eq. 1.64 holds

$$g^{(2)}(0) = 1 + \frac{\Delta n^2 - \bar{n}}{\bar{n}^2}.$$  

(1.77)
1.7 Photon Statistics

Taking $\Delta n$ and $\bar{n}$ for Laser respectively thermal light, the results known from the classical theory are obtained

$$g^{(2)}(0) = 1 \quad \text{Laserlicht},$$
$$g^{(2)}(0) = 2 \quad \text{thermisches Licht}.$$

1.7.4 Antibunching

On the contrary to the classical description, eq. 1.77 allows values of $g^{(2)}(0)$ lesser than 1. In the classical description, the following equation holds:

$$g^{(2)}(0) = 1 + \frac{\langle \Delta I^2 \rangle}{\langle I \rangle^2} \geq 1 \quad (1.78)$$

For a photon number state $|n\rangle$ without any fluctuations $\Delta n$ one obtains

$$g^{(2)}(0) = 1 - \frac{1}{\bar{n}} < 1. \quad (1.79)$$

This effect, which is called antibunching, cannot be explained using classical fields. Qualitatively the photons are emitted completely regularly and therefore do not show any fluctuations $\Delta n$. While photons are bosons, this behaviour is rather fermionic. An example for the generation of nonclassical light is the resonant fluorescence of a single atom. Dependent on the value of $g^{(2)}(0)$ three different classes of light can be distinguished:

- $g^{(2)}(0) > 1$ Chaotic/thermal light, the large fluctuations cause the photons to reach the detector in bunches. This bunching effect can also be described by classical theory.
- $g^{(2)}(0) = 1$ Laser, the photons are completely uncorrelated.
- $g^{(2)}(0) < 1$ Nonclassical light, the photons are more regularly spaced than the photons in a laser beam. If the emission is completely regular the effect is called antibunching.
Figure 1.18: Photon emission for (a) a thermal source, (b) a Laser (c) a nonclassical source. The vertical lines symbolize a detected photon.
1.8 Gaussian Beams

One of the properties of a laser beam is its low divergence. A laser beam propagating along the $z$-axis behaves similar to a plane wave along this direction, but is closely bound in transverse direction.

Most HeNe-Lasers emit in the TEM$_{00}$-mode, where the abbreviation TEM denotes Transverse Electro Magnetic mode. This mode is also called Gaussian fundamental mode and is therefore denoted as TEM$_{00}$. In this mode, for the distribution of the electric field assuming axial symmetry and propagation in $z$-direction holds

$$E(\rho, z) = E_0 \frac{\omega_0}{\omega(z)} e^{-(\frac{\omega(z)}{\omega_0})^2} e^{\frac{i z k e^2}{2 R(z)}} e^{i \left(\frac{k z - \frac{1}{2 \tan\left(\frac{z}{z_0}\right)}}{z_0}\right)}$$

with the transverse coordinate $\rho = \sqrt{x^2 + y^2}$, the beam waist $\omega_0$, the beam radius $\omega(z)$, the Rayleigh length $z_0$ and the wavefront curvature radius $R(z)$. These parameters are examined more closely in the following.

The first exponential function in eq. 1.80 describes the transverse amplitude distribution and has a Gaussian profile. The factor in the second exponential function describes the spherical curvature of the wavefronts, the factor in the last exponential the phase along the $z$-axis.

![Figure 1.19: Drawing of the beam radius $\omega(z)$ of a Gaussian beam with along the propagation direction $z$ with beam waist $\omega_0$, Rayleigh length $z_0$ and curvature radius $R(z)$.](image)

The confocal parameter $b$ respectively the so-called Rayleigh zone $2z_0$ is given by

$$b = 2z_0 = \frac{2\pi \omega_0^2}{\lambda}.$$  \hfill (1.81)

In the Rayleigh zone the beam radius $\omega(z)$ increases by a factor of $\sqrt{2}$. In this zone the beam divergence has its lowest value. At $z = \pm z_0$, the wavefront has its largest curvature respectively the lowest radius $R(z_0) = 2z_0$. For arbitrary $z$ holds

$$R(z) = z \left(1 + \left(\frac{z_0}{z}\right)^2\right).$$  \hfill (1.82)

In the Rayleigh zone for $z \ll z_0 : R(z) \simeq \infty$, in the far field $R(z) \simeq z$. 
The beam waist \( \omega_0 \) is obtained as

\[
\omega_0^2 = \frac{\lambda z_0}{\pi}.
\]  

(1.83)

The beam waist only depends on \( \lambda \) and the Rayleigh length \( z_0 \). The development of the beam curvature in \( z \)-direction is described by

\[
\omega(z) = \omega_0 \sqrt{1 + \left( \frac{z}{z_0} \right)^2}.
\]

(1.84)

As one can see from the equations above the TEM\(_{00}\)-mode is completely described by the two parameters \((\omega_0, z_0)\). In a simplified way the intensity distribution of a Gaussian mode perpendicular to the direction of propagation can be written as

\[
I(\rho, z) \propto |E(\rho, z)|^2 = I_0 \left( \frac{\omega_0}{\omega(z)} \right)^2 e^{-2\rho^2/\omega(z)^2}.
\]

(1.85)

This corresponds to a Gaussian distribution. \( I_0 \) is a constant dependent on the beam intensity. The diameter of the Gaussian beam is described as the area, where the intensity drops below \( \frac{1}{e^2} \) respectively \( \approx 13\% \).

![Figure 1.20: Transverse intensity profile of a Gaussian TEM\(_{00}\)-mode dependent on the beam radius \( \rho \) for an arbitrary value of \( z \).](image)

If a Gaussian laser beam is focused by a lens, the beam radius in the focal plane is given by

\[
\omega_0 = \frac{\lambda f}{\pi \omega(z)},
\]  

(1.86)

where \( \omega(z) \) is the beam radius at the position of the lens, \( f \) the focal length and \( \omega_0 \) the beam radius (beam waist) in the focus.
2 Experimental Setup

In the following, the experimental setup is described. Section 2.1 covers the setup for the measurement of the temporal second-order correlation function. On contrary to the historical HBT experiment, only one detector instead of two is used. This is feasible because the temporal resolution of the detector is much smaller than the coherence time of the source. Therefore one detector is sufficient to measure $g^{(2)}(\tau)$ of the source. In section 2.2 the setup for the measurement of the spatial second-order correlation function $g^{(2)}(r_1, r_2)$ is examined. Because spatial correlation-measurements require to measure coincident photons at two spatially separated points, two detectors are needed. In the following, the hardware and software used for the experiment is discussed.

For all parts of the experiment a HeNe-Laser $^1$ is used. As can be seen in fig. 2.1 and 2.2 directly behind the laser a telescope is assembled in the beam path. The telescope widens the beam to enlarge the confocal area, where the beam diameter is nearly constant. The telescope is adjusted in a proper way to position the confocal area at the optical bench, where the lenses can be assembled, therefore the beam diameter is nearly constant at the position of the lenses. The widened beam is attenuated and after some reflections passes the lens by which the beam is focused onto a rotating ground glass disc. The disc transforms the coherent light of the laser to pseudothermal light (cf. sec. 1.2.3). This light is separated into two beams by a beam splitter and guided to the two photomultiplier by fibers $^2$, which detect the photons of the two beams (cf. fig. 2.1 and 2.2). The photons detected by the photomultipliers are processed as described in sec. 2.4.2, afterwards the signals are read in and time-stamped by a data acquisition card $^3$. The PC buffers the data and provides them to the individual programs. One program is sued to calculate the autocorrelation function, which is proportional to $g^{(2)}(\tau)$ and calculates the photon statistics, a second program measures the coincidences of the two detectors to calculate $g^{(2)}(r_1, r_2)$.

$^1$Melles Griot 05-LHP-121, Output power 2 mW, $\lambda = 632.8$ nm, beam diameter at output $0.59 \pm 5\%$ mm
$^2$Hamamatsu H7360-02, dark count rate < 50 s$^{-1}$, quantum efficiency (633 nm 2%, pulse width 10 ns)
$^3$National Instruments NI PCIe-6320, X-Series DAQ; temporal resolution 10 ns
2.1 Setup for the measurement of the temporal second-order correlation function

As described above, for the measurement of $g^{(2)}(\tau)$ and the photon statistics $p(n, T)$ only one detector ($D_1$) is used.

The measurement program „FP-Photonenstatistik, zeitliche Korrelation“ can calculate the current photon count rate, the temporal second-order correlation function $g^{(2)}(\tau)$ and the photon statistics $p(n, T)$. From $g^{(2)}(\tau)$ information about the coherence and photon statistics of the light source can be obtained.
2.2 Setup for the measurement of the spatial second-order correlation function

Figure 2.2: Setup for the measurement of the spatial second-order correlation function. Similar to the setup for the measurement of the temporal second-order correlation functions, only with two detectors, one of them movable laterally to the beam. The speckle pattern caused by the rotating disc is spatially limited by different circular apertures, separated at the 50/50-beamsplitter and sent to optical fibers which guide it to detectors $D_1$ and $D_2$. The measured single count rates and the coincidence count rate are grabbed by the computer.

Fig. 2.2 shows the experimental setup for the measurement of the spatial second-order correlation function $g^{(2)}(r_1, r_2)$. Unlike for the measurements of the temporal second-order correlation function both of the two detectors are used. One of them is mounted on a motorized translation stage movable laterally to the beam propagation direction. The signals are processed by logic circuits as described in sec. 2 and 2.4.2 and by the measurement software „FP-Photonenstatistik, räumliche Korrelation“. This setup serves to measure the spatial second-order correlation function of circular pseudo-thermal sources of different sizes. Afterwards, the diameter of the sources is calculated from the functional form of $g^{(2)}(r_1, r_2)$. This reproduces the historical HBT-experiments for the measurement of the diameter of stars (cf. sec. 1.5.1).
2.3 Characteristic curve of the ground glass disc motor

Because thermal light has a typical coherence time of $10^{-15} - 10^{-9}\text{s}$ which is below the temporal resolution of common photodetectors, it is feasible to use a pseudothermal source as described in sec. 1.2.3. Before starting the measurement it is important to know the velocity of the rotating disc at the position of the laser spot on the disc, respectively the rotation frequency and the distance between rotation axis and laser spot. With this additional information, using eq. 1.17 one can calculate the theoretical coherence time of the pseudothermal source. The calibration of the rotation frequency of the disc as a function of the supply voltage $\nu(U)$ gives the following relation:

$$\nu(U) = 0,3120(9) \frac{1}{\text{Vs}} U - 0,073(6) \frac{1}{\text{s}},$$

(2.1)

where the error is given by

$$\Delta \nu(U) = \sqrt{(0,0009 \cdot U)^2 + (0,006)^2 + ((U \cdot 0,01 + 0,02) \cdot 0,312)^2}.$$ 

(2.2)

![Figure 2.3: Characteristic curve $\nu(U)$ of the ground glass disc motor.](image-url)
2.4 Detection electronics

2.4.1 Digital camera

The first step of the experiment is to measure the beam diameter $D$ at the position of the optical bench for the lenses and to analyse the speckle pattern behind the ground glass disc. This is done using a digital camera whose chip has a resolution of $1280 \times 1024$ pixels. The pixel size is $5.3 \times 5.3 \mu m^2$. The analogue-to-digital-converter has a resolution of $2^8 = 256$ bits, therefore, the brightness of every pixel takes a value between 0 and 255. According to the producers data sheet, the pixels respond linearly with increasing illumination.

The camera is delivered with the software uEye Cockpit which allows to take pictures and to save them as bmp-files. In this data format, the values of the pixels are saved as matrix, which has $1280 \times 1024$ entries in the case of the pictures taken in the experiment. The free software FreeMat allows to transform the bmp-file to a matrix in coordinate format and to save it as an ASCII-file, which can be processed with GnuPlot or a similar program for further analysis.

2.4.2 Signal processing and coincidence circuit

The electric signals of the two detectors $D_1$ and $D_2$ are so-called TTL-pulses. For the measurement of the temporal second-order correlation function, only detector $D_1$ is in use and no coincidences are measured. For the measurement of the spatial second-order correlation function both detectors $D_1$ and $D_2$ are used and their coincidences are detected. In a first step, the TTL-pulses need to be converted to NIM-pulses, because the coincidence unit for the spatial $g^{(2)}$-measurement can only process such pulses. The discriminator

---

4Data Vision UI-1240SE-M-GL, CMOS-Sensor

5TTL (Transistor-Transistor Logic); a logical 0 (1) corresponds to $< 0,8 \text{ V}$ ($> 2,2 \text{ V}$)

6NIM (Nuclear Instrument Module); a logical 0 (1) corresponds to $> -0,8 \text{ V}$ ($< -0,8 \text{ V}$)
widens the NIM-pulses to a length of $T_K = 40\text{ ns}$. This defines the coincidence window of 40 ns. The discriminator has two output channels, one leads directly to the computer, the other one to the coincidence unit. The signal sent to the computer has to be converted back to a TTL-pulse to allow the data acquisition card to read the pulses. If the coincidence unit recognizes two pulses from $D_1$ and $D_2$ as coincident, a signal is sent to the computer, which has first to be converted to a TTL-pulse, too. Two signal are recognized as coincident, if the two widened pulses of the detectors have an overlap. The probability of such an event depends on the pulse length $T_K$. If a pulse width too short is chosen, the coincidence rat is very low, for a pulse length too high uncorrelated photons can be detected as coincident, therefore the coincidence rate is too high. Fig. 2.5 schematically shows a coincident event as the overlap of two pulses.

Figure 2.5: Schematic view of two overlapping pulses of pulse length $T_K$, which causes a coincident event.
2.5 Software

2.5.1 Measurement program „FP-Photonenstatistik“

For all correlation measurements, a self-developed program „FP-Photonenstatistik“ is used, which analyses the data of the data acquisition card and saves the data. The program is launched using the link on the desktop. After starting the program, the following selection-menu appears: (cf. Fig. 2.6).

![Selection menu of the measurement program.](image)

The button for the measurement one wants to execute has to be selected, afterwards the window containing the selected subprogram opens.

**Temporal correlations and photon statistics**

If the button „zeitliche Korrelation“ is selected, the program for the measurement of the temporal second-order correlation function $g^{(2)}(\tau)$ and the photon statistics $p(n, T)$ (cf. fig. 2.7) appears in a new window.

![Program for the measurements of the temporal correlations and photon statistics using one detector.](image)
When a measurement is launched, \( g^{(2)}(\tau) \) and the photon statistics are acquired simultaneously. If „Zählrate“ is activated, the current count rate of \( D_1 \) in Hz is displayed. If „\( g^{(2)}(\tau) \)“ is selected, the \( g^{(2)}(\tau) \) data are displayed, if „Photonenstatistik“ is selected, the photon statistics, i.e. the number of photons per time interval is displayed. By clicking the button „Neu“ a new measurement cycle is started. If a measurement cycle is started, the number of the current measurements already acquired is shown in „Anzahl Messungen g2“. One measurement corresponds to the temporal separation between two photons. By clicking the „Stop“-button, the current measurement is stopped. To save the data one has to select the „Speichern“-button. By clicking this button, both the \( g^{(2)}(\tau)\)-data and the photon statistics are saved in separate txt-files. When all measurements are completed one should exit the program by selecting „Fenster schließen“.

Spatial correlation measurements

If the button „räumliche Korrelation“ is selected, the program for the measurement of the spatial second-order correlation function \( g^{(2)}(r_1, r_2) \) (cf. fig. 2.8) is launched in a new window.

If a measurement cycle is started, the single-count rates of the two detectors, the coincidence rate and \( g^{(2)}(r_1, r_2) \) are acquired simultaneously. By activating „Zählrate“ the current single-count rates and the coincidence rate in Hz as well as the normalized spatial second-order correlation function are displayed. By selecting „g1 Detektor 1“, „g1 Detektor 2“, „Koinz. Rate“ and „\( g^{(2)} \)“ the data for the single-count rates, the coincidence count rate or \( g^{(2)} \) are displayed graphically. Detector 2 is mounted on an electronically driven translation stage. The parameters for that translation stage can be adjusted in the „Schrittmotorsteuerung“-menu before starting a new measurement cycle. The parameter „Schrittrgröße“ defines the stepsize and „Messzeit“ the measurement period per data point, with a larger period, the statistics of the data point are enhanced. A measurement cycle is started by selecting the „Messung Starten“-button and stopped using „Messung Stoppen“. To save the data, one has to select „Daten Speichern“. The data for one measurement cycle are saved in one single txt-file. After finishing all the measurements, the

Figure 2.8: Program for the measurement of the spatial coherence.
program can be exited by selecting „Fenster schließen“.

### 2.5.2 uEye Cockpit

*uEye Cockpit* is the software to control the digital camera. It is started via the link „uEye Cockpit“ on the desktop.

![Software to control the digital camera](image)

When the window shown in fig. 2.9 is opened, one has to start the camera by selecting the button red-framed in fig. 2.9. Additionally, one has to access the „Shutter“ menu via the button green-framed in fig. 2.9 and to switch to „Rolling Shutter“. This selects the readout-mode for stationary objects and suppresses the background.

### 2.5.3 FreeMat

FreeMat is an *Open Source* development environment which is partially compatible to the commercial program MATLAB. In the analysis of this experiment it is used to process the data saved as bmp-files and save them in a txt-file.

In the experiment, greyscale-pictures are grabbed which have a depth of $2^8$ bits, i.e. every pixel has an integer value between 0 and 255. For the analysis it is important to know the frequency of every value in a picture and to convert it to a format which can be processed e.g. with GNUPlot.

In the following, the usage of the FreeMat-scripts provided for the data analysis is explained.

The first script stored in the directory „Skripte“ on the desktop is named „COO-Matrix FreeMat“. In line 1 and 2, two bmp-files containing the background and the beam profile called „Streulicht.bmp“ respectively „Laserstrahl.bmp“ in fig. 2.10 are read-in. The file in line 1 is the picture of the background with the shutter of the laser closed, the file in line 2 a picture of the laser beam. In line 4 the pixel values of the background are subtracted from those of the picture of the laser beam. In line 6 a value of 1 is added to every pixel, because the value 0 is not taken into account by FreeMat. In line 8 the value $v$ is assigned to every entry $i, j$ of the matrix. In line 10 the matrix is saved in the ASCII-output file named „DurchmesserMatrix.txt“ (red in fig. 2.10), where the first column contains the line...
index of the matrix, the second column of the matrix the column index and the third line
the corresponding value. This file can be processed with the Gnuplot-script „3D_Gaussfit“.

```
1   H=imread('Streulicht.bmp')
2   A_raw=imread('Laserstrahl.bmp')
3   A=2*A_raw
4   A=A/2
5   i=1
6   [i,j,v]=find(A)
7   COO=[i,j,v]
8   save -ascii -tab COOmatrixMatrix.txt
```

Figure 2.10: COO-Matrix FreeMat-script to convert the bmp-file to a txt-file, in which
the pixel information is saved as a matrix in coordinate form (line, column, value).

The second script, which is stored in the directory „Skripte“, too, is named „Histogramm
FreeMat“. In line 1 and 2 two bmp-files containing the background and the speckle pattern
called „Streulicht.bmp“ and „Speckle.bmp“ in fig. 2.11 are read in. The first one is the
picture of the background with the laser shutter closed, the second one the picture of the
speckle pattern behind the ground glass disc. In line 4 the pixel values of the two pictures
are subtracted. In line 5 the number of pixels with a certain value is counted, in line 10
the information are saved in the file „SpeckleHistogramm.txt“ (red in fig. 2.11). The first
column of this file contains the value, the second column the absolute frequency of this
value. This can be plotted with, e.g., GnuPlot.

```
1   H=imread('Streulicht.bmp')
2   H=imread('Speckle.bmp')
3   B=H
4   [b,b]=hist(C(:,1),unique(C(:,1)))
5   A=histcount(b)
6   c=histcount(C(1,:))
7   save -ascii -tab SpeckleHistogramm.txt
```

Figure 2.11: COO-Matrix FreeMat-Script to convert the bmp-file to a txt-file containing
the absolute frequency of a every pixel value.

In the following, the usage of the scripts is explained step by step. Fig. 2.12 shows the
2.5 Software

graphical user interface of FreeMat. As a first step, the path to the files have to be adapted (red in the figure). Afterwards, the scripts COO-Matrix FreeMat and Histogramm FreeMat can be loaded by selecting the blue-highlighted button. A window containing the script opens. There, the names of the input and output files have to be adapted. Afterwards, the whole script is copied and pasted in the command line 2.12 highlighted green. This starts the execution of the script, and the output file is saved.

2.5.4 GnuPlot scripts

In the directory „Skripte“ on the desktop, GnuPlot-scripts containing the fit functions required for the data analysis are store. The path and the name of the input file have to be adapted in the scripts. If the fits don’t work properly, the initial values of the fit parameters have to be adapted. The scripts have to be completed by additional commands to label the axes and set the output terminal.

Scripts are provided for a Lorentz (Lorentz.txt) respectively Gaussian fit (Gauss.txt) to check the kind of pseudothermal light. Both are used to calculate the coherence time and the value of the temporal correlation function for $\tau = 0$, too. Furthermore, there are three scripts to determine the underlying probability distribution of the photon statistics $p(n,T)$ (Poisson.txt, Bose-Einstein.txt, Boltzmann.txt). For the determination of the beam diameter $D$ of the laser beam at the position of the optical bench the script 3D_Gaussfit.txt is provided. For the calculation of the diameter of the circular apertures via $g^{(2)}(r_1,r_2)$ there is the script Lochblende.txt.
3 Experimental Tasks

Before starting with the experiment, create a new directory and save all your data there.
After doing the experiment, you can copy the data and take them home to do the analysis.
Important: Select meaningful file names!

3.1 Laser Attenuation

After switching the HeNE-laser on, there are strong intensity fluctuations in the beginning.
These fluctuations fade after about 15 minutes almost completely. Remember to switch
on the laser in time.

ATTENTION!!! The power supply for the photomultipliers must only
be switched on while doing the measurements. When opening the box lid
to change apertures, attenuators or lenses or when doing other operations
which can cause an illumination of the photomultipliers too strong the
power supply must be switched off, else the photomultipliers can be dam-
aged. The maximal allowed countrate is about 10 MHz (i.e. 10 million
photons per second).

ATTENTION!!! For security reasons, a HeNe-Laser with only 2 mW
output power is used. Therefore the laser falls in class 3R. Direct expo-
sition to the eye can cause damages. Therefore, directly before the laser
an attenuator with OD = 0.5 is used to attenuate the power to < 1 mW.
Nonetheless, following security rule applies:

NEVER LOOK DIRECTLY IN THE LASER BEAM!

Before starting the measurements you should get a sense of the neutral density filters
required to attenuate the HeNe-lasers with an output power of (2 mW) to a detector
countrate of $D_1$ of about 400,000 photons/s (400 kHz). The degree of attenuation of a
neutral density filter is described by the optical density (OD) and calculates as

$$\text{OD} = -\log(T) \quad \text{bzw.} \quad T = 10^{-\text{OD}}, $$

(3.1)

where $T$ denotes the transmittance. The relation between OD and $T$ is shown in tab. 3.1.
The transmittance can be derived from the Beer-Lambert law of absorption

$$I(\alpha) = I_0 e^{-\alpha d} \quad \rightarrow \quad T = \frac{I(\alpha)}{I_0} = e^{-\alpha d}. $$

(3.2)

where $I_0$ is the output intensity, $\alpha$ the absorption coefficient and $d$ the thickness of the
absorbing material. The optical density OD also is proportional to the product of $\alpha$ and $d$.
3 Experimental Tasks

<table>
<thead>
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<th>OD</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(%)</td>
<td>79</td>
<td>63</td>
<td>50</td>
<td>40</td>
<td>32</td>
<td>25</td>
<td>10</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 3.1: Optical density and corresponding transmittance

- Calculate the attenuation required to obtain a countrate of about 400 kHz. Consider, how many photons per second are emitted by a Laser with a power of 2 mW.

3.2 Properties of the pseudothermal light source

3.2.1 Measurement of the beam diameter

As you could see in the drawing of the experimental setup (cf. fig. 2.1) the laser beam is expanded using a telescope. For the analysis you need the beam diameter at the translation stage. Therefore you have to measure the beam diameter there using a digital camera (see sec. 2.4.1). With the camera software *uEye Cockpit* you can save the images as bitmaps.

![Figure 3.1: Beam profile imaged with a digital camera](image)

Take five pictures at different positions along the translation stage to calculate the beam diameter $D$ and its error. The translation stage lies inside the Rayleigh zone of the Laser beam, where the beam divergence has its lowest value (cf. sec. 1.8). For the analysis use the FreeMat-script „\texttt{COO FreeMat}“ to transform your image into a matrix containing the pixel coordinates and pixel values and which is provided at the computer at the experiment. The textfile can now be used to calculate the beam diameter via a fit of a 2-dimensional Gaussian function (GnuPlot Script „\texttt{3D_Gaussfit.txt}“).

Using the beam diameter $D$ the beam waist radius $\omega_0$ in the focus of a lens can be calculated (cf. sec. 1.2.3). This information will later be needed to calculate the theoretical coherence time of the pseudothermal light source.

- Take an exposure of the background while the laser shutter is closed. For the next exposures use the same settings (room illumination, exposure time etc.) as for this exposure.

- Take five exposures at five different positions along the translation stage. Calculate the beam diameter and perform an error analysis.
3.3 Temporal coherence and photon statistics

- Calculate the beam waist radius and its error after focusing the beam with lenses of the focus \( f = 10, 15, 20, 25 \) and \( 30 \) cm.

3.2.2 Analysing the photon statistics of the pseudothermal source via the speckle pattern

In this part of the experiment the speckle pattern which is generated by the coherent illumination of the rotating ground glass disc is analyzed. Place the camera in the mount labeled 'Kamera Speckle' behind the glass disc and take a picture of the speckle pattern for the unfocused laser beam. Also, take a background image with closed laser shutter for the use in the analysis. Take care to have the same settings (room illumination, exposure time etc.) for the background image as for the speckle image. You can convert the images to a format which can be processed by programs like gnuplot or Excel with the script „Histogramm FreeMat“. The textfile generated by the script gives you the absolute frequency of a specific intensity in the speckle pattern. At this data you can fit a distribution function (GnuPlot Scripts „Poisson.txt“, „Bose-Einstein.txt“, „Boltzmann.txt“).

- For which reason a distance from the ground glass disc to the camera of more than \( 20 \) cm was chosen (cf. sec. 1.2.3)?

- Take a background image with the laser shutter closed. Use the same settings (room illumination, exposure time etc.) for the following images.

- Take a image of the stationary speckle pattern, i.e. with static disc, and analyse the intensity distribution of the speckle pattern. Does it correspond to the photon statistics of a thermal light source? Discuss your results and consider which statistics to expect for a changing speckle pattern, i.e. for a rotating disc.

3.3 Temporal coherence and photon statistics

Remove the camera from the beam path. Start the measurement software „FP-Photonenstatistik“, with the link on the desktop and select the option „zeitliche Korrelation““. The mode of operation of the program is discussed in sec. 2.5.1..

3.3.1 Background measurement

As in the case of the digital camera, the background light has to be taken into account for the photomultipliers, too. At first the countrate with the laser shutter closed is to be read out, by this the background can be measured. With the box lid closed, the background countrate should be \(< 100 \) Hz and can therefore be neglected in the next measurements. Important: You don’t have to start a \( g^{(2)} \)-measurement, only observe the countrate! The countrate must not exceed \( 10 \) MHz, else the photomultipliers can take damage!

- Check the countrate with the lid closed and vary the room illumination. Open the lid slowly while the room illumination is switched on and discuss your observations.

3.3.2 Measurement of the second order temporal correlation function for laser light

In the beginning of the experiment you calculated the attenuation needed to reduce the countrate to \( 400 \) kHz. Now measure \( g^{(2)}(\tau) \) and the photon statistics for the attenuated
laser beam by detecting about 1.000.000 time intervals (‘zeitliche Abstände’) between photons (cf. sec. 2.5.1).

- Did the calculated attenuation corresponded to the value needed in reality? If not, what could be the reason for this discrepancy? A qualitative discussion is sufficient.

- Measure $g^{(2)}(\tau)$ for the attenuated laser beam. Are your expectations concerning $g^{(2)}(\tau)$ confirmed experimentally?

- Which probability distribution describes the photon statistics of laser light? Test it with an appropriate fit.

- Calculate the variance $\Delta n^2$ from the photon statistics. Discuss your results.

### 3.3.3 Measurement of the second order temporal correlation function for pseudothermal light

Put the ground glass disc into the beam path to assemble your pseudothermal light source. For the next measurements, the lenses are used to focus the laser beam on the glass disc. Optimize the distance from the lens to the ground glass disc via the speckle pattern using eq. 1.16. For every lens log the distance $r$ between the center of the disc and the laser spot. For every measurement, adjust the attenuation to achieve countrates of about 400 kHz. Now measure $g^{(2)}(\tau)$ and the photon statistics by measuring about 1.000.000 time intervals (‘zeitliche Abstände’) between photons (cf. sec. 2.5.1).

- Decide via a $g^{(2)}(\tau)$-measurement if the pseudothermal source simulates collision- or Doppler-broadened light (GnuPlot scripts „Lorentz.txt“, „Gauss.txt“).

- Assemble a lens with a focal length of $f = 20$ cm and measure the correlation function for four different rotation frequencies of the ground glass disc. Form this calculate the corresponding coherence time $\tau_{c,\text{mess}}$ with an appropriate fit and compare it with the theoretical coherence time $\tau_{c,\text{theo}}$ (s. Eq. 1.17) dependent on the rotation speed of the disc.

- Calculate and plot the simulated power spectrum for one of the coherence times $\tau_c$ measured above using the Wiener-Khintchine Theorem (cf. sec. 1.4.3). Be careful to scale the $x$-axis in an adequate way.

- replace the lens used before by the ones with $f = 10, 15, 25$ and 30 cm and measure the correlation function for each of them for one constant rotation frequency. Use an adequate fit to obtain $\tau_{c,\text{mess}}$ and compare the values to the theoretical values $\tau_{c,\text{theo}}$ dependent on the focal length $f$.

- Which probability distribution describes the photon statistics of the pseudothermal light source? Use an adequate fit to confirm this. Discuss your results.

- Calculate the variance $\Delta n^2$ via $g^{(2)}(0)$ (cf. eq. 1.77) for all your measurements and compare them to the values you obtain from the photon statistics. Discuss your results.
3.4 Measurement of the spatial correlation of different light sources

Close the subprogram „zeitliche Korrelation“ and select the option „r"aumliche Korrelation“. The operating mode is explained in 2.5.1. For every measurement a complete set of data, consisting of the single countrates of the two detectors, the coincidence rate and the spatial second order correlation function is taken.

Remove the lens $f_3$ from the measurements before (not one of the telescope lenses!) and the ground glass disc out of the beam path. Mount a circular aperture in the beam path directly after the position of the ground glass disc. Take care that the aperture is illuminated homogeneously. You can check this by looking at the diffraction pattern of the circular aperture, an Airy-disc, with the camera and adjusting the aperture to obtain a homogeneous, rotation symmetrically, pattern. Take a measurement with the coherently illuminated aperture. After that, place the ground glass disc before the aperture and repeat the measurement for the incoherently illuminated aperture. Repeat the two measurements for two other circular apertures of your choice. You save time by taking the two sets of measurements for coherent respectively incoherent illumination for one aperture before changing the aperture. As parameters for the stepper motor use a step size of $100\,\mu\text{m}$ and a measurement of $1\text{s}$ per step.

- Measure the countrates $D_1$ and $D_2$ and $g^{(2)}(r_1, r_2)$ for three different apertures with coherent illumination. Discuss your results.
- Measure the countrates $D_1$ and $D_2$ and $g^{(2)}(r_1, r_2)$ for three different apertures with incoherent illumination. Discuss your results and compare it with the results for coherent illumination.
- Use the van Cittert-Zernike Theorem to calculate the diameters of the apertures (cf. eq. 1.57).

3.5 Measurement of the spatial correlation of a double slit

Mount the double slit mask in the beam path at the position of the apertures. Take care that the double slit is illuminated homogeneously. You can check this by looking at the diffraction pattern of the double slit with the camera and adjusting the slit mask to obtain a homogeneous and symmetric pattern (single central maximum, symmetric side maxima of equal intensity). You have to place the camera shortly in front of the beam splitter to justify the far-field condition of the double slit.

Take a correlation measurement with the coherently illuminated slit mask. After that, place the rotating ground glass disc before the slit mask and repeat the measurement for the incoherently illuminated double slit. As parameters for the stepper motor use a step size of $25\,\mu\text{m}$ and a measurement of $1\text{s}$ per step.

- Measure the countrates $D_1$ and $D_2$ and $g^{(2)}(r_1, r_2)$ for the double slit with coherent illumination. Discuss your results qualitatively.
- Measure the countrates $D_1$ and $D_2$ and $g^{(2)}(r_1, r_2)$ for the double slit with incoherent illumination. Discuss your results qualitatively and compare it with the results for coherent illumination.